Models of Set Theory II - Winter 2017/2018

Prof. Dr. Peter Koepke, Ana Njegomir Problem sheet 10

Problem 1 (6 Points)**.** Suppose that $\kappa \leq \lambda \leq \mu$ are uncountable regular cardinals. For $Y \subseteq [\mu]^{<\kappa}$, the *projection* of Y to λ is defined as

$$Y_\lambda = \{y \cap \lambda \mid y \in Y\}.$$

For $X \subseteq [\lambda]^{<\kappa}$, the *lifting* of X to μ is defined as

$$X^\mu = \{x \in [\mu]^{<\kappa} \mid x \cap \lambda \in X\}.$$

Prove the following statements:

(a) If S is stationary in $[\mu]^{<\kappa}$, then S_λ is stationary in $[\lambda]^{<\kappa}$.

(b) If C is club in $[\mu]^{<\kappa}$, then C_λ contains a club in $[\lambda]^{<\kappa}$.

(c) If S is stationary in $[\lambda]^{<\kappa}$, then S^μ is stationary in $[\mu]^{<\kappa}$.

Problem 2 (5 points)**.** Suppose that $\mathbb{P}$ is a σ-closed forcing. Let κ be an uncountable cardinal in M and suppose that $p \Vdash_{\mathbb{P}}^{M}$ "$\dot{C} \subseteq [\check{\kappa}]^{\check{\omega}}$ is club" for some $p \in \mathbb{P}$ and a $\mathbb{P}$-name $\dot{C}$. Prove that there is a club $D \subseteq [\kappa]^\omega$ in M such that for all $x \in D$ there are sequences $\langle p_n \mid n \in \omega \rangle$ and $\langle x_n \mid n \in \omega \rangle$ such that for all $n \in \omega$, $p_{n+1} \leq_{\mathbb{P}} p_n \leq_{\mathbb{P}} p$, $x_{n+1} \supseteq x_n$, $p_n \Vdash_{\mathbb{P}}^{M} \check{x}_n \in \dot{C}$ and $\bigcup_{n\in\omega} x_n = x$.

Problem 3 (4 points)**.** Let $\mathbb{P}$ be a σ-closed forcing and let κ be an uncountable cardinal. Prove that every stationary subset of $[\kappa]^\omega$ remains stationary in every $\mathbb{P}$-generic extension of M.

Problem 4 (5 points)**.** Suppose that $\mathbb{P}$ satisfies the countable chain condition and let G be $\mathbb{P}$-generic over M. Prove that for every uncountable λ, every closed unbounded set $C \subset [\lambda]^\omega$ in $M[G]$ has a subset $D \in M$ that is closed unbounded in M.

Please hand in your solutions on Monday, December 18 before the lecture.